

## Measurements of the speed of sound in air–water flows

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### Abstract

Pressure transducer measurements, recorded while calibrating two conductivity-based, void fraction meters, show how the propagation velocity of pressure waves in bubbly air–water flows varies with void fraction. The results for void fractions between 0 and 0.5 indicate that, as expected, the propagation velocities in an air–water mixture are considerably lower than the speed of sound in either component; they agree well with prediction methods based on the mixture density and bulk modulus. © 1997 Elsevier Science S.A.

*Keywords:* Air–water flows; Void fraction; Sound

### 1. Introduction

It is well known that the presence of small amounts of gas in a liquid reduces significantly the velocity at which pressure (or sound) waves can travel through the mixture. Whilst calibrating a conductivity probe, designed to measure void fraction, pressure transients were recorded which enabled the speed of sound in a bubbly air–water mixture to be related to the void fraction.

### 2. Apparatus and results

The measurements were made in the vertical test section shown in Fig. 1. The tube was assembled from flanged sections of transparent acrylic plastic with a bore of 32 mm and a wall thickness of 3 mm. At the top of the test section a large bore plastic ball valve was attached to a rigid support, the tube was suspended below this valve and supported by pipe clamps. The test section inlet consisted of a brass housing with air and water connections and a tapping which accommodated a 0 to 3.5 bar gauge pressure transducer. Above the ball valve approximately 0.7 m of 38 mm bore copper pipe rose to a bend, which returned the air and water mixture to the storage tank; the tank also acted as a separator. Two void fraction meters were mounted in the test section; these were similar to those described by Ma et al. [1], but were operated with temperature compensation, as described by Costigan and Whalley [2]. Their outputs and that of the inlet pressure transducer were recorded on a computer via a high speed data-logger.

Solenoid operated valves were fitted to the air and water inlet pipes and connected through a microswitch on the ball valve to a power supply. When the ball valve handle was moved towards the closed position, the solenoid valves were closed in less than 3 ms; it took about 0.1 s after this to close the ball valve completely.

A typical test consisted of setting up a steady two-phase flow in the test section. Inlet pressure, void fraction and temperature signals were recorded at either 100 Hz or 1000 Hz for at least 20 s before the ball valve was closed. Data recording was stopped after 40 s had elapsed. The trapped volumes of liquid and gas indicated the mean void fraction of the flow and this was compared with the void meter signals. The meters were calibrated for void fractions between 0 and 1.0 and were found to be most accurate in the bubbly flow region (where mean void fractions could be as high as 0.45), with a tendency to indicate slightly higher average void fractions than were measured by the quick-closing valve or level-swell techniques.

The pressure transducer output records from 1 s before to 4 s after the ball valve closure are shown in Fig. 2. When the inlet valves closed, a rarefaction wave travelled from the bottom of the tube to the top, where it was reflected as a pressure wave. The amplitude of successive reflections decayed rapidly, as the energy of the wave was dissipated. After the initial transient, the pressure increased gradually, owing to a slight leak from the air solenoid valve. This had no effect on the measured void fraction and the change in mean pressure over the duration of the transient was negligible.

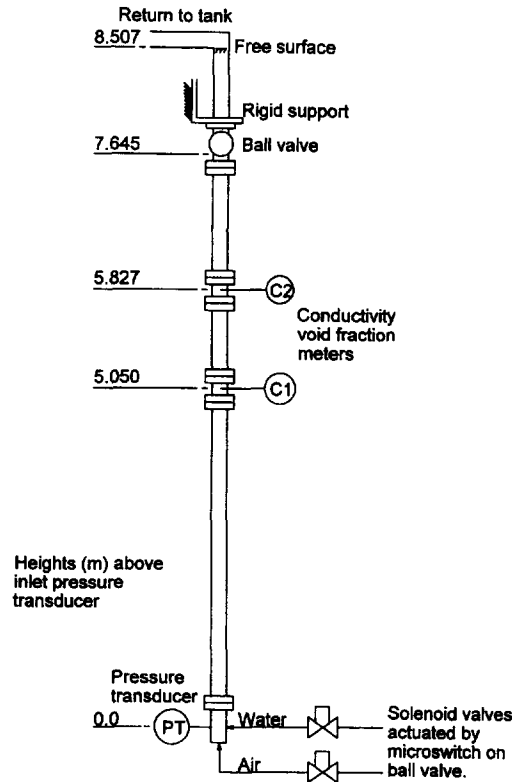


Fig. 1. Arrangement of the vertical air–water test section.

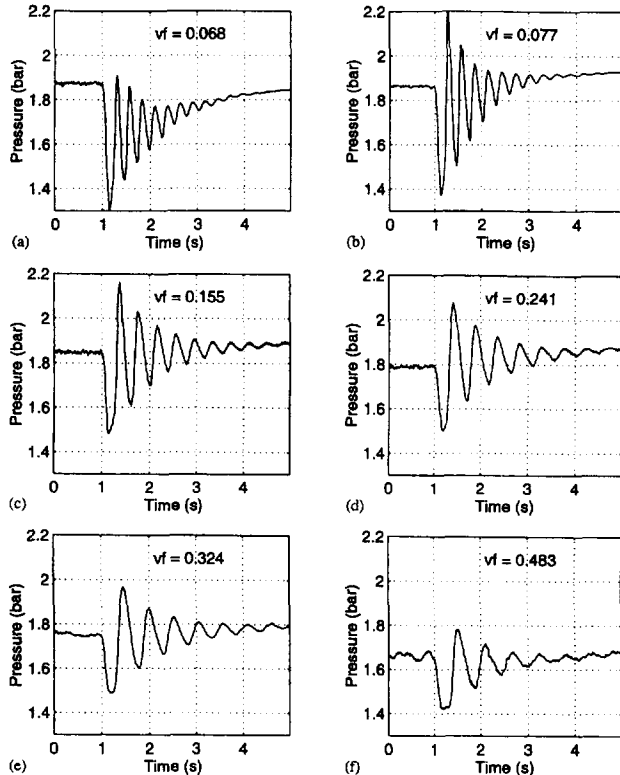


Fig. 2. Transient pressures after inlet valve closure: (a)  $c = 57.99 \text{ m s}^{-1}$ , (b)  $c = 52.92 \text{ m s}^{-1}$ , (c)  $c = 38.93 \text{ m s}^{-1}$ , (d)  $c = 32.13 \text{ m s}^{-1}$ , (e)  $c = 28.92 \text{ m s}^{-1}$ , (f)  $c = 26.31 \text{ m s}^{-1}$ .

The mean wave speed was determined by measuring the time taken for a number of complete cycles of pressure, and

dividing the distance travelled ( $2 \times \text{number of cycles} \times \text{tube length}$ ) by it. The resultant values are shown in Fig. 2 and also in Table 1.

The six traces (a) to (f) were recorded at a constant water flow, equivalent to a superficial velocity  $U_{ls}$  of  $0.974 \text{ m s}^{-1}$ , and increasing superficial air velocities  $U_{gs}$  as shown in Table 1. These values were selected to ensure that the discrete bubble flow pattern was maintained throughout. At air velocities higher than those of trace (f) (in which void waves were beginning to form) the occurrence of spherical cap bubbles and slugs damped out the pressure transients almost immediately.

Void fractions were measured using the higher of the two conductance void meters (C2). Costigan and Whalley [2] have shown that these meters tend to overestimate slightly the true void fraction in bubbly flow; therefore the void fraction  $\alpha$  was also calculated from the superficial velocities and the bubble rise velocity  $U_b$  thus:

$$\alpha = \frac{U_{gs}}{U_{ls} + U_{gs} + U_b} \quad (1)$$

where  $U_b$  is given by the equation [3]

$$U_b = 1.53 \left[ \frac{g(\rho_l - \rho_g)\sigma}{\rho_l^2} \right]^{1/4} \quad (2)$$

Both measured and calculated void fractions are shown in Fig. 3, where they are compared with theoretical values from the equation

$$c = \left\{ [\rho_l(1 - \alpha) + \rho_g\alpha] \left[ \frac{\alpha}{np} + \frac{(1 - \alpha)}{K_1} \right] \right\}^{-1/2} \quad (3)$$

The derivation of Eq. (3) can be found in [4]. The theoretical line was plotted using the isothermal polytropic index ( $n = 1.0$ ) in Eq. (3) as this was found to give the best representation of our data as well as that of Karplus cited by Gouse and Brown [5]. However, the adiabatic index ( $n = 1.4$ ) gives the correct value for air when  $\alpha = 1.0$ . This apparent discrepancy is due to the fact that the thermal capacity of a low pressure bubbly mixture of air and water is dominated by the properties of the liquid and, since the gas

Table 1  
Measured and calculated quantities; water superficial velocity  $0.974 \text{ m s}^{-1}$

	Trace						Fig. 4(a)
	a	b	c	d	e	f	
Air superficial velocity ( $\text{m s}^{-1}$ )	0.059	0.082	0.174	0.308	0.483	1.039	0
Void fraction from I	0.046	0.063	0.124	0.201	0.283	0.459	0
Measured void fraction	0.068	0.077	0.155	0.241	0.324	0.483	0.020*
Measured speed of sound ( $\text{m s}^{-1}$ )	58	52.9	38.9	32.1	28.9	26.3	261.5

\*Measured by the level swell technique.

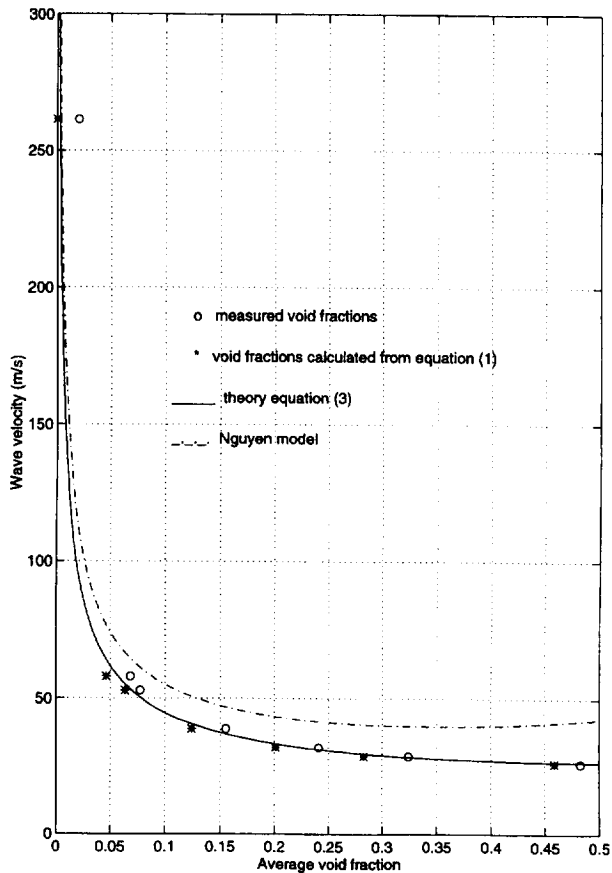


Fig. 3. Speed of sound for air–water mixtures.

is dispersed in the liquid, it will behave approximately isothermally.

The highest wave velocity recorded was derived from the pressure transient shown in Fig. 4(a); these data were obtained by leaving the ball valve at the top of the tube open and closing the water solenoid valve. The pressure wave was then reflected from the free surface at the bottom of the return elbow (see Fig. 1), rather than the ball valve. The water at the test section inlet had a void fraction of zero. Operating experience indicated that it was particularly important to have no leaks under these conditions. Accordingly, the air inlet solenoid valve was disconnected and the connection was sealed off, and the water solenoid valve was renewed. A short length of 6 mm bore plastic tube, attached to the test section as a drain, caused unacceptable damping of the pressure waves and it was replaced by copper fittings.

The transient shown in Fig. 4(b) was obtained under the same operating conditions as that in Fig. 4(a); the only difference was that the ball valve was closed, in the same way as the tests illustrated in Fig. 2. The speed of sound measured in this case was only  $52.2 \text{ m s}^{-1}$ .

### 3. Discussion

The agreement between data and the predictions of Eq. (3) in Fig. 3 is very good, although the wave velocity is not

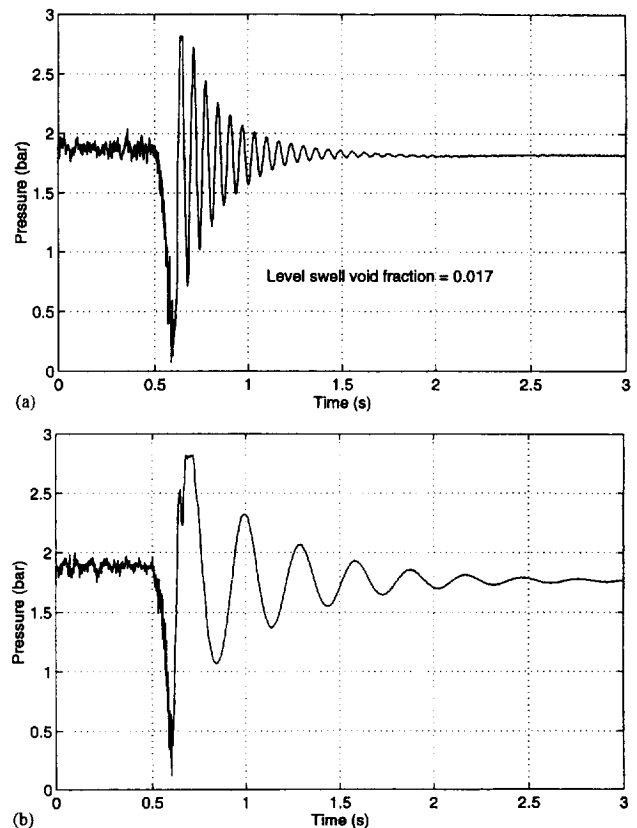


Fig. 4. Speed of sound for zero inlet void fraction: (a) no air flow, ball valve open,  $c = 261.5 \text{ m s}^{-1}$ ; (b) no air flow, ball valve closed,  $c = 52.2 \text{ m s}^{-1}$ .

particularly sensitive to variations of void fraction above 0.2. When the elasticity of the pipeline was taken into consideration, as suggested by Wylie and Streeter [6] among others, the effect on the results of Eq. (3) was indistinguishable on the axes of the diagram. However, at a void fraction of zero the speed of sound in pure water was reduced from  $1484.5 \text{ m s}^{-1}$  to  $495.3 \text{ m s}^{-1}$  by its inclusion.

Legius et al. [7] have made speed of sound measurements in bubbly flows with void fractions up to 0.15. They claim good agreement with the homogeneous model of Nguyen et al. [8], therefore Nguyen's predictions are also included in Fig. 3. If the isentropic index of  $n = \gamma = 1.4$  is used in Eq. (3), there is close agreement with Nguyen's model over the range of Legius' data. However, our data and those cited by Gouse and Brown [5] are more closely described by the isothermal version of Eq. (3).

The maximum measured wave velocity of  $261.5 \text{ m s}^{-1}$  corresponds to a void fraction of 0.003, according to Eq. (3). The velocity for zero void fraction is  $495.3 \text{ m s}^{-1}$ , therefore some air must have come out of solution in the water during the passage of the rarefaction wave. The water was held in the storage tank at atmospheric pressure, as it was pumped around the circuit it was exposed to air in the return line and the tank, therefore it is likely that it was saturated with air at this pressure. Henry's law shows that water can hold 2% by volume of air under these conditions. In addition, a small amount of entrained air could have been carried into the pump

and some of this would also enter solution as the water was pressurized. The mean inlet pressure before the water valve was closed was 1.87 bar, thus it is possible that the water entering the tube contained up to 3.7% of dissolved air. When the water was circulating normally its pressure fell to 1.0 bar at the top of the test section, thus it is likely that small bubbles of air were continually being evolved in the test section. When the rarefaction wave passed, the pressure fell to below 0.1 bar absolute, providing the opportunity for much more air to come out of solution. It is difficult to know exactly how much air will come out of solution under these circumstances, since the process is time dependent, but it cannot be more than the maximum value quoted above of 3.7% by volume, and it is likely to be much less than this.

The conductivity void meters did not give reliable quantitative values at these low void fractions owing to a slight zero offset, therefore the average void fraction was deduced from measurement of the collapsed liquid level after the transient, which was 8.337 m. Assuming that the two-phase mixture level was the height of the free surface (8.507 m), the average void fraction was calculated to be  $(1 - 8.337/8.507) = 0.02$ ; this is the measured value plotted in Fig. 3. This value is likely to be an underestimate, since the two-phase level would have risen above the bend when the pressures were low, and some water would have been lost as it overflowed back to the tank.

The wave speeds for zero inlet void fraction (Fig. 4(a) and (b)) were consistently different, depending on whether the exit ball valve were open or closed. The initial depressurization was very similar in both cases: the ragged pressure traces suggest that air was coming out of solution during this period. The passage of the first pressure wave in Fig. 4(b) was retarded by a number of reductions in pressure, and the wave frequency was subsequently reduced. It is not known whether these falls in pressure corresponded to air being re-dissolved in the closed volume of the tube as the pressure rose, but it is generally agreed that gases come out of solution owing to depressurization much more rapidly than they return when pressure is re-applied, therefore it is unlikely that all of the air would be re-dissolved. Another possibility is that the pressure reductions represent reflections from large voids in the body of the liquid.

In either event Eq. (3) indicates that a wave speed of  $52.2 \text{ m s}^{-1}$  corresponds to a void fraction of 0.07. We have argued above that void fractions will not have been higher than 0.037, but that they could have been higher than 0.02; therefore a value as high as 0.07 requires explanation. It has already been pointed out that the ball valve closed an estimated 0.1 s later than the solenoid valve; it is therefore possible that the ball valve closed at or near the point of lowest pressure in the tube. Since the lowest pressure was about 0.1 bar, the air density would be low and this would give rise to a considerable increase in the volume of the two-phase mixture. Closing the ball valve under these conditions would 'freeze' a high void fraction in the tube, since no more liquid could enter. When the ball valve was left open, however, oscillations of

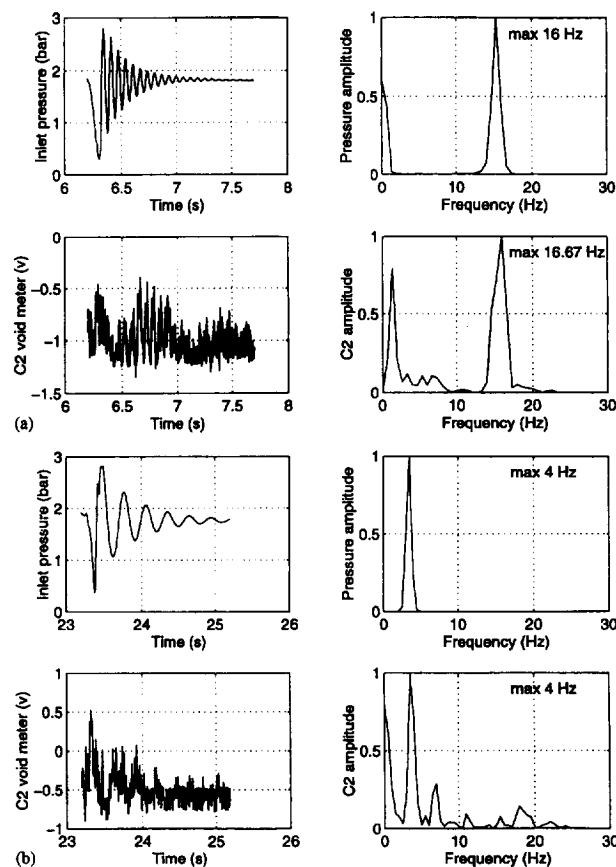


Fig. 5. Comparison of pressure and void meter signals: (a) no air flow, valve open; (b) no air flow, valve closed.

the free surface level could accommodate void fluctuations with a small loss of liquid from the tube, but no significant change in mean void fraction or liquid level.

The void meter signals are compared with the pressure traces in Fig. 5(a) and (b) for the open and closed valve cases respectively. The power spectral density (PSD) of each signal is shown in the right-hand column. The data of Fig. 5 indicate convincingly that the pressure variations create void fluctuations of the same frequency in both cases.

#### 4. Conclusions

The velocity of sound in bubbly air–water mixtures, with void fractions approaching 0.5, has been measured by analysis of the pressure transient which follows sudden closure of the inlet valves.

The results agree closely with the theoretical predictions of Eq. (3), which gives the speed of sound as the square root of the ratio of the mixture bulk modulus to the mixture density. The Nguyen model is less accurate.

At zero inlet void fraction, air comes out of solution with the water during the initial depressurization. The subsequent velocity of sound depends on whether the exit ball valve has

been left open, in which case it will be about  $260 \text{ m s}^{-1}$ , or has been closed, when it will be about  $52 \text{ m s}^{-1}$ . The reason for this significant difference is not clear.

## 5. Nomenclature

$c$	Speed of sound
$g$	Gravitational acceleration
$n$	Polytropic index
$p$	Absolute pressure
$K_1$	Bulk modulus of the liquid
$U_b$	Bubble rise velocity
$U_{gs}$	Superficial gas velocity
$U_{ls}$	Superficial liquid velocity
$\alpha$	Void fraction
$\rho_l$	Density of liquid
$\rho_g$	Density of gas
$\sigma$	Surface tension

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